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A Rescaled Range Analysis of Random Events[1]

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Abstract — The rescaled range statistical analysis was applied on sets of random numbers to demonstrate its potential in studying various types of biases and the presence of periodical features. The data were generated by electronic random number generators in psychokinesis tests. According to the theory of Hurst, the rescaled range of independent random data is proportional to the square root of their number. In data which are not independent, the fractional Brownian motion model of Mandelbrot is useful in modeling their time series as persistent or anti-persistent. A weak predominantly persistent type of fractional Brownian motion in the data indicated a bias which could not be distinguished from chance fluctuations after comparison with computer simulated data. The basic steps for the application of this method, the variety of information it can provide and its limitations are discussed. The method provides a relatively simple, yet robust, technique for studying anomalies in random events.

Keywords: anomalies — fractional Brownian motion — Hurst exponent — periodicities — psychokinesis — rescaled range analysis

1. Introduction

Science is often confronted with puzzles which stay unsolved for long periods of time, one of these being the Hurst effect (Feder, 1988). The effect has been initially observed in sequences of records in time, $x(t)$, of natural phenomena such as river discharges, which were expected to be independent over long periods of time. Hurst, in his analysis, first transformed the natural records in time into a new variable $X(t,N)$, the so-called accumulated departure of the natural record in time in a given year t ($t = 1, 2, ..., N$), from the average, $\bar{x}(t)$, over a period of N years (Hurst, 1951; Korvin, 1992). The transformation follows the formula

$$X(t,N) = \sum_{i=1}^{t}\left(x_i - \bar{x}_N\right) = \left\{\sum_{i=1}^{t} x_i\right\} - t\bar{x}_N \tag{1.1}$$

[1]Part of this work was presented at the 1996 Euro SSE conference. Request for reprints should be addressed to: Fotini Pallikari, Physics Department, University of Athens, Zografos, Panepistimiopolis, Athens 157 84, Greece.

Then, he introduced the rescaled range R/S, in which the range $R(N)$ is defined by

$$R(N) = \max_{1 \le t \le N} X(t, N) - \min_{1 \le t \le N} X(t, N) \tag{1.2}$$

and the standard deviation $S(N)$ by

$$S(N) = \left\{ \frac{1}{N} \sum_{t=1}^{N} \left[x_t - \bar{x}_N \right]^2 \right\}^{1/2} \tag{1.3}$$

With the use of the dimensionless rescaled range, Hurst was able to compare natural phenomena of various kinds. He found, then, that the natural phenomena he studied followed the empirical law

$$R/S = (\alpha N)^H \tag{1.4}$$

where the Hurst exponent H — called K by Hurst — was more or less symmetrically distributed about a mean 0.73 with a standard deviation (SD) of about 0.09. Although this value was slightly overestimated due to the small number of available data for certain phenomena, Hurst was able to reproduce the relation (1.4) by using simulations of biased random events. He derived the relation (1.5) below, which the rescaled range parameter should follow for truly unbiased events, and was able to show that his own simulations of unbiased random events obeyed it:

$$R/S = (\pi N/2)^{1/2} \tag{1.5}$$

This relation was also derived and discussed by Feller (1951) and later confirmed by Feder (1988) by a computer simulation of random events.

Hurst's simulations of independent random processes (1951) were done by tossing n (= 10) coins N (= 1000) times and taking the random variable x to be the number of heads minus the number of tails. It is worth pointing out, as it will be discussed later, that the random variable in this simulation can be represented by the net displacement of a particle undergoing a typical random walk on a line where the particle moves in steps of equal length either to the left or to the right at equal probability. Feder has, however, shown that the position in time, $x(t)$, of a particle that walks at random with steps of unit length on a line becomes asymptotically an ordinary Brownian motion with a Gaussian distribution of step lengths for time scales much larger than the time between steps and for distances much larger than the unit step length. The variable $X(t, N)$ of Equation (1.1) represents, in fact, the position of the particle which makes steps $x(t)$ away from the origin.

Mandelbrot later introduced a generalized form of the Brownian motion model, the fractional Brownian motion, fBm, to model the Hurst effect (Mandelbrot & Van Ness, 1968; Mandelbrot, 1982). In the fBm model the Hurst exponent, as shown in relation (1.4), is a real number in the range $0 < H < 1$. There are three types of generalized fBm: (a) the persistent, for values of H in the range $0.5 < H < 1$, (b) the anti-persistent, for $0 < H < 0.5$, and (c) the case $H = 0.5$ which corresponds to the independent white noise processes of ordinary Brownian motion.

According to Mandelbrot, to be anti-persistent is to tend to turn back toward the point one came from or, in terms of the random walk picture, to diffuse slower than in the ordinary Brownian motion. Any increasing trend in the past makes a decreasing trend in the future more probable, and *vice versa,* the strength of this anti-correlation depending on how much lower than 0.5 the H parameter is. The other type of fBm, the persistent, implies that the increments' persistence is maintained over longer periods of time, depending on the Hurst exponent value. If some time in the past there is a positive increment — *i.e.* an increase — it is more likely that there will be an increase in the future, while a decreasing trend in the past implies the likelihood of a decreasing trend in the future. In the random walk language, one tends to diffuse faster away from the origin than in a Brownian motion.

The correlation between past and future increments, C, of the accumulated departure from the mean was estimated by Feder as[2]

$$C = 2^{2H-1} - 1 \tag{1.6}$$

which is notably independent of time. The correlation C takes positive values for persistent fBm, negative values for anti-persistent fBm and is zero for independent random events. Since the correlation C is independent of the lag time t in time series, or the number of data N when in a computer simulation of random events, the fBm predicts an infinitely long correlation in persistent and anti-persistent fBm's.

According to the fBm model, records in time of natural phenomena exhibit in general a persistent type of fBm. The presence of such infinitely long correlations is in conflict with what is observed in statistical records in time of physical systems. Any correlations within records in time of physical systems tend to die off as their temporal separation increases. It is true, on the other hand, that the Hurst exponent drops off as the size of data set on which it is applied

[2]Relation (9.16) of Feder (1988).

increases, as will be discussed later, to reach asymptotically a value in agreement with what Equation (1.4), or (3.1) below, estimates.

The rescaled range analysis is considered as a robust method for investigating the presence of correlations in random events, whether there are significant statistical deviations from chance or not, or even if the data obey Gaussian statistics or not. Triggered by the unusual fBm behavior of natural phenomena, this work sets out to investigate how the fBm model applies to events generated by random number generators in psychokinesis tests. In the psychokinesis tests that will be reported here, the pre-stated conscious intention of the operator with regard to the test outcome is introduced as a psychophysical parameter. The aim of the present analysis is, therefore, to investigate the type of the generalized fractional Brownian motion which best fits the statistical behavior of random numbers generated in psychokinesis.

2. Experimental

The random numbers presented in this work were generated with the use of two Schmidt electronic random number generators (RNG), machines A & B. A standard statistical analysis on the data has been published elsewhere (Pallikari-Viras, 1997) in the light of the balancing effect idea (Pallikari-Viras, 1993, 1995, 1998), where details of the RNG's operation can be found. Machine A operated on the basis of a combination of white noise from a semiconducting diode and a quasi-random bit generated by an 80C39 microprocessor (Schmidt & Braud, 1993). The statistical distribution of numbers generated by this machine is expected to be centered about zero with a standard deviation $\sigma = 133.1$ according to calibration tests (Schmidt & Stapp, 1993). Machine B operated on the basis of a quasi-random algorithm with a PIC16C57 microprocessor using a DL2416 EEPROM memory chip for storing the data. The algorithm for converting the random bits generated with machine B into random numbers is identical to Feder's (1988) computer simulation of the Hurst coin tossing experiment. The random numbers in the present experiment represent the difference between the number of heads (1's) and tails (0's) in tossing 100 coins N times. The data statistics of these random numbers obey a binomial distribution of mean zero and standard deviation

$$\sigma = 2\sqrt{\frac{1}{2} \times \frac{1}{2} \times 100} = 10$$

which is twice the standard deviation of the distribution of the heads, or tails, alone. The data on which the Hurst analysis will be applied constitute four sets of PK random numbers, their combinations — as will be explained further — and one randomness test.

The chronological order by which the tests were done is indicated by their number, in Table 1. Tests # 1 and 2 were done with machine A and tests # 3, 4 and 5 were done with machine B. The identification "intention" in Table 1 describes the pre-stated intention of the operator in favor of a certain binary outcome, whereas "no intention" describes a no pre-stated intention during the data generation for any of the test outcomes. Within a test both conditions were alternatively applied in regular intervals. In the randomness test, however, data were generated only under a no-intention condition.

3. Data Analysis

The Hurst exponent H was estimated from the slope of the straight line which represents the graph log(R/S) *vs.* log N, as in Equation (1.4):

$$\log(R/S) = \log \alpha + H \log N \tag{3.1}$$

In a test of a total number of data N_t, the ratio R/S was estimated according to relations (1.1)–(1.3), as follows. The set of N_t numbers is divided into υ groups, each group containing $N = N_t/\upsilon$ data, where υ takes values from 1 to N_t such that $N \geq 4$ is also an integer. One also includes more data points in the graph, to improve the quality of the data fitting, by selecting such integers υ for which only a very small proportion of the whole data (four data or less) is excluded from the analysis. The quantity R/S is estimated for each of the v groups, and their average, $\overline{R/S}$, is then plotted against N in a log–log graph. The standard deviation of the mean $\overline{R/S}$ for the respective N is represented as error bars in the graphs. For instance, if the total number of data is $N_t = 1024$ this set is divided into two groups each of $N = 512$ data and the two independent values of R/S as well as their average $\overline{R/S}$ and $\log(\overline{R/S})$ are estimated, for $N = 512$. Then, the 1024 numbers are divided into four groups of 256 data each and $\log(\overline{R/S})$ is estimated for $N = 256$. The subdivision of data continues until one gets 256 groups of four data each and $\log(\overline{R/S})$ are as usual estimated for $N = 4$. The quantity $\log(\overline{R/S})$ is also estimated for the whole data set. A typical $\log(\overline{R/S})$ *vs.* log N graph is shown in Figure 1. It refers to all the data generated by machine A under no-intention condition. The parameters α and H in Equation (3.1) estimated from the linear regression analysis of the data are shown in Table 1. The R/S analysis was applied on the data obtained on each of the two conditions in every one of the four tests, plus on the randomness test, as well as on the new sets formed by their merged data as follows:

(a) The numbers generated under both conditions within each test were merged and analyzed as one set marked in Table 1 as "intention & no intention," tests # 1, 2, 3, and 5. It represents all the data generated within a test.

TABLE 1
Statistical and Rescaled Range Analysis Parameters

Number of Test	Condition	Number of Data	Mean	SD	Hurst Exponent[a]	α[a]
	intention	1024	+6.99	133.24	0.496 ± 0.012	1.05 ± 0.03
1	no intention	940	–7.05	134.50	0.606 ± 0.016	0.77 ± 0.03
	intention & no intention	1964	+0.27	134.01	0.588 ± 0.013	0.65 ± 0.02
	intention	482	–4.47	135.12	0.645 ± 0.018	0.57 ± 0.02
2	no intention	482	+3.77	135.52	0.492 ± 0.015	1.10 ± 0.04
	intention & no intention	964	–0.35	135.31	0.545 ± 0.010	0.88 ± 0.02
	intention	1506	+3.32	133.91	0.566 ± 0.010	0.83 ± 0.02
1&2	no intention	1422	–3.38	134.91	0.586 ± 0.005	0.83 ± 0.02
	intention & no intention	2928	+0.07	134.42	0.574 ± 0.009	0.79 ± 0.02
	intention	257	+1.61	9.96	0.551 ± 0.033	0.80 ± 0.06
3	no intention	257	+0.16	9.89	0.414 ± 0.021	1.61 ± 0.07
	intention & no intention	514	+0.88	9.94	0.573 ± 0.028	0.78 ± 0.06
4	randomness test	1026	+0.09	10.01	0.503 ± 0.008	1.07 ± 0.02
	intention	256	–0.16	10.12	0.73 ± 0.05	0.44 ± 0.05
5	no intention	256	+0.94	10.03	0.47 ± 0.04	1.13 ± 0.09
	intention & no intention	512	+0.39	10.08	0.66 ± 0.03	0.53 ± 0.03
	intention	513	+0.70	10.07	0.71 ± 0.03	0.43 ± 0.02
3&5	no intention	513	+0.55	9.97	0.49 ± 0.02	1.08 ± 0.05
	intention & no intention	1026	+0.63	10.01	0.62 ± 0.01	0.64 ± 0.02

[a] Parameters obtained from the linear regression of Equation (3.1), above $N = 32$.

(b) Data generated under the same condition over the two tests with the same machine were also merged. This combination is marked as 1 & 2 (machine A) and 3 & 5 (machine B) in the column titled "Number of Test."

(c) The whole data generated by each of the two machines, A or B, regardless of condition was analyzed as one larger set of data. It is marked in Table 1 as "intention & no intention" under "condition" and "1 & 2 or 3 & 5" under the "# of test" column.

The randomness test data were not involved in the merging because they were not generated under the same binary intention condition protocol.

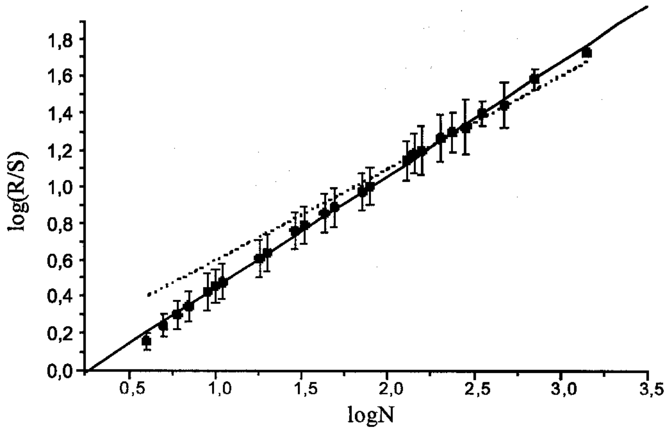

Fig. 1. A typical Hurst graph plotting log() *vs.* log N, of all data generated by machine A, test # 1 & 2 under no-intention condition. The straight line is a linear regression fit to the data. The dotted line plots relation (1.5) in the text.

4. Results and Discussion

The R/S analysis done on the individual nine data sets (two machines, two tests with each machine, two conditions within each test, plus one randomness test) of random numbers presented here fits Hurst's empirical equation (1.4). The Hurst exponent estimated from this analysis indicates the presence of weak biases within most of the data sets of varying size (from $N = 256$ to $N = 1026$) as well as within the ten additional ones of larger size ($N = 512$ to $N = 2928$) which resulted from the merging of the initial eight data sets in various combinations shown in Table 1.

The H parameter varies strongly with N if small subsets of data are considered, Figure 2, to asymptotically reach a constant value, not necessarily within a given whole of data. The fBm character in the set of data is contained collectively in all subgroups of size N in the Hurst analysis procedure, but not adequately within the small size ones as their statistics are not representative in this case. Therefore, to involve subsets of small size, N, in the data fitting would overestimate the H parameter and the right size N_c above which the data fitting is perfomed must be decided. Feder estimated the H parameter of a set of 50,000 computer simulated random data for $N > 20$ by using the same complicated fitting procedure we have applied here. The Hurst exponent presented in Table 1 is estimated in a data fitting for $N \geq 32$. In view of the small size of some data sets, this may have overestimated the H parameter. It should be pointed out, however, that the value thus estimated agrees with the H

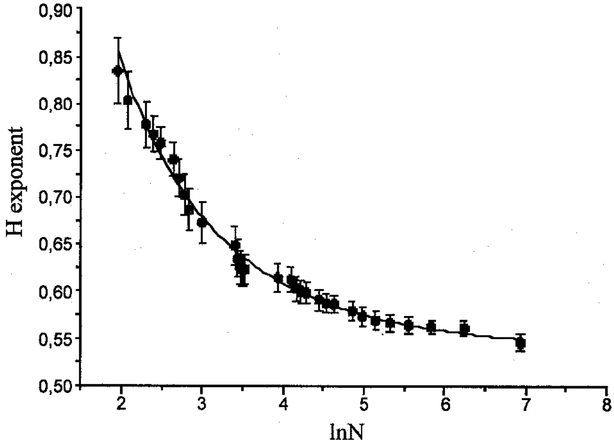

Fig. 2. A plot of the H exponent against the natural logarithm, ln N, of the size of data set, N, for the data of test # 1 under intention condition. The continuous curve represents an exponential fit to the data.

parameter that would have been obtained from extrapolation if the data set was as large as a few hundred thousand units, Figure 2.

When weak biases are detected in the Hurst analysis of small data sets it is not at first easy to decide whether their origin is a biasing agent or if the H parameter was overestimated in the data fitting procedure. Feder has found, for instance, that if he included all possible subsets in the analysis of the 50,000 computer simulated random data the H parameter was overestimated by 3%. An appropriate cut-off N_c for the data fitting has to be decided. However, the effect of the N_c value in the estimation of the H parameter is less serious, if very large size data sets are analyzed with no periodic features in them.

It is advisable in any case to draw conclusions on the fBm character of the random data not by studying individual experimental data sets, but by comparing them with empirical distributions of H parameters. To fix a certain cut-off N_c calibration data or computer simulations of the random data generation process can be used. We have simulated the algorithm of machine B and generated 1000 data sets of 1024 data each twice for two different seed numbers with the use of a Borland Pascal 7.0 compiler. The average H parameter was then estimated in a data fitting above an N_c, for each of the two different seed numbers, changing N_c from 32 to 200 as in Table 2. Changing the seed number in the pseudo-random process did not seem to affect the H parameter. All the estimated H parameters were, within one standard deviation of the mean H, consistent with the theoretical expectation of an independent process. The average H, however, seems to depend on N_c. A lower N_c yields a higher H expo-

TABLE 2
The Hurst Exponent of Computer Simulated Random Number Sequences for Different Cut-off Sample Sizes N_c and Seed Numbers

N_c	Seed Number	H	SD
32	101010	0.53	0.05
50	101010	0.52	0.07
100	101010	0.51	0.10
150	101010	0.50	0.13
200	101010	0.50	0.14
32	983	0.53	0.05
50	983	0.53	0.07
100	983	0.52	0.10
150	983	0.51	0.12
200	983	0.51	0.14

nent. Higher cut-offs estimate H exponents as expected by independent data, but having relatively high standard deviations. The variance of the H parameters is in some cases relatively large and these observed individual runs of biased data should be considered as the result of chance fluctuations.

The weak biases observed in most of the individual experimental data sets generated by the two machines, are of the persistent type with the exception of the no-intention test # 3 which presented an anti-persistent type of bias. The randomness test is, for the H value, consistent with the theoretical expectation from independent data. The standard deviation of the H parameter estimated in the data fitting process varies from test to test. In order to give an overall estimate of the H exponent across tests, the standard deviation should be taken into account in estimating a weighted average[3] (Barlow, 1989). The weighted average of all the nine H parameters is = 0.520 with SD = 0.005. If the randomness test is excluded from the averaging then, 34 = 0.532 with SD = 0.009 (n = 8). In particular, the four intention data sets yield a weighted average H parameter, H = 0.549, with SD = 0.009, whereas the four no-intention data sets yield a weighted average H = 0.515 with a sample SD = 0.009 in good agreement with an independent random process (or, H = 0.508 , SD = 0.006 if the randomness test is included in the averaging). A summary of these results is presented in Table 3.

Comparison of the results of Tables 2 and 3 shows no difference between experimental data and the simulated data for cut-off N_c = 32 and the predominant persistence found in the data could be assigned to chance fluctuations. It is not absolutely clear, however, at this point if the estimated weak persistence in the simulated data is due solely to the selection of the cut-off N_c or to periodical features in the quasi-random process itself. A large number of calibration data would be helpful in this case. To conclude, on the basis of the data presented

[3] with a variance .

TABLE 3
The Hurst Exponent of the Psychokinesis Experimental Data

Number of Data Sets	Weighted Average H[a]	SD[b]
9 (intention + no int. +random test)	0.520	0.005
8 (int.+ no int.)	0.532	0.009
4 (intention)	0.549	0.009
4 (no-intention)	0.515	0.009
5 (no int. + random test)	0.508	0.006

[a]According to footnote 3 and for $N_c = 32$.
[b] According to footnote 3.

here and although the average H parameter of intention data is higher than that of no-intention data, one has to sustain that the weak persistence is due to statistical fluctuations and not related to the psychophysical parameter involved in the tests.

5. Studying Periodicities in the Random Data

Periodical features in the random data may be observed on the Hurst graph showing as deviations from linearity around the corresponding period. In the Hurst graph of sunspot activity, for instance, an undulation is observed at about $N = 11$ years (Feder, 1988). This deviation from linearity results from an irregular change in the rescaled range and standard deviation of the data due to the periodical feature. The presence of periodicities in the data interfere with the correct estimation of the H parameter. One cannot rely, therefore, on the slope of the Hurst graph alone to get a complete picture of the biases in the data, but the graph as a whole should be considered.

There were undulations in many of the Hurst graphs of data reported here. In general this could have resulted from periodicities related either to the psychophysical variable introduced, or to the biases inherent in the random generation process. Since there was no clear connection between the psychophysical parameter introduced in this work and the presence of periodicities, we shall have to assume that their presence is due to the latter case. Although this is not a problem that can be solved here, the advantages of this application to the investigation of present periodicities are presented. An extended study on this topic and the interpretation of the periodical features is intended for future work.

In Figure 3, for instance, the Hurst graph of all the data generated by machine A is shown. An undulation is observed at about $N = 450$. This periodical feature is not present in the graph of Figure 1, where the subset of the data generated by the same machine under the no-intention condition is plotted. However, the Hurst graph of the intention subset of data, generated by machine A, shows a very weak undulation at about $N = 450$, Figure 4. If this periodicity were more prominent it would have implied a possible connection with

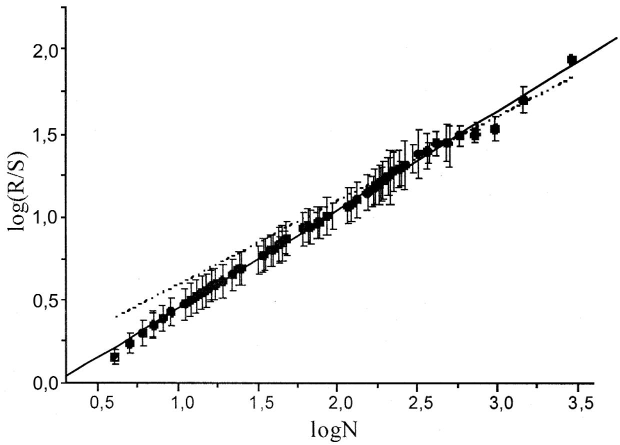

Fig. 3. The Hurst graph of the whole data generated by machine A. The straight line is a linear regression fit to the data. The dotted line plots relation (1.5) in the text.

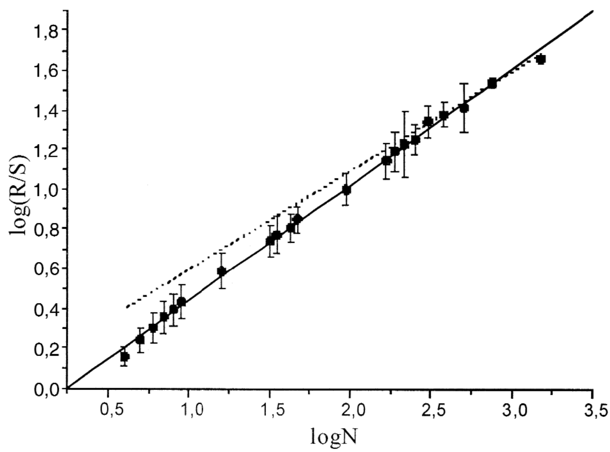

Fig. 4. The Hurst graph of all data generated by machine A under intention condition. The straight line is a linear regression fit to the data. The dotted line plots relation (1.5) in the text.

the psychophysical parameter introduced. It should be reminded, however, that the majority of data sets presented here are not showing significant statistical deviations of data averages from chance, with the exception of test #3 intention at a 0.01 level of significance. This is an individual case and together with the other data it may simply be the result of chance fluctuations in the random process.

In an attempt to shed some light on the issue of periodicities, one of us (E.B.) programmed the computer to introduce a variety of biases into the original randomness test data. There were three types of biases introduced, a sinusoidal bias, a point bias, and a data correlation bias. In the sinusoidal bias, a term
x was added to the original data of a size which varied according to the expression

$$x = A \sin\left(\pi \frac{N}{B}\right) \quad (5.1)$$

that is, having a period of $N = 2B$ and an amplitude A, Figure 5, where $B = 20$ in Figure 5a and 80 in Figure 5b, while $A = 10$ in both cases. The effect of the sine bias on the original data is seen to start at about a data size equal to a quarter of a period, while structures appear in the graph at data sizes equal to multiples of the period. A small period sine bias introduces anti-persistence in the original data, while a long period introduces persistence. The Hurst analysis of the biased data is shown in Table 4. Additional information about the value of one standard deviation of the data fitting procedure and the respective correlation estimated, indicating the quality of fitting, are presented.

As an example of the effect of a point bias, a constant number of size 50 added after every 30 (Figure 6a) and every 10 (Figure 6b) of the original data is shown. The actual effect of the point bias on the original data was only to introduce anti-persistence and that was observed for a large range of lengths of

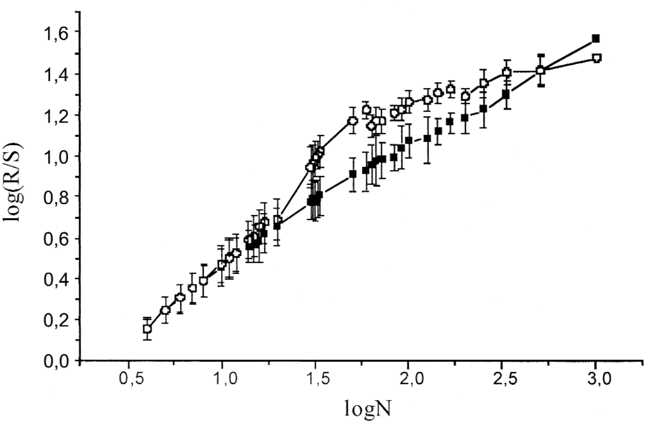

Fig. 5a.

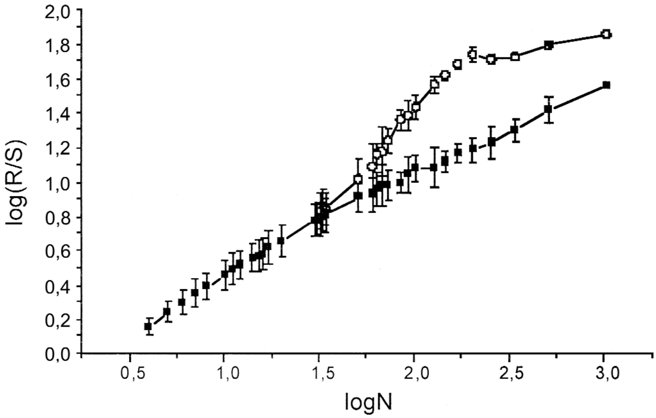

Fig. 5b.

Fig. 5. The Hurst graph of randomness test data (■) and artificially biased data (○)with an added sinusoidal term of amplitude 10 according to relation (5.1) for (a) a period 40 and (b) a period 160.

TABLE 4
Effect of Artificial Biasing on the Hurst Exponent of a Sequence of Random Numbers

File	H	SD	R
Original random data	0.503	0.008	0.996
Biased. Sine-20-10	0.314	0.023	0.953
Biased. Sine 80-10	0.791	0.067	0.937
Biased. Point 10-50	0.499	0.009	0.997
Biased. Point 30-50	0.480	0.011	0.995
Biased. Correlated	0.995	0.019	0.997

data intervals and magnitudes of bias. Finally, the original data x were transformed according to

$$x'_n = \sum_{i=1}^{n} x_i - n\bar{x} \tag{5.2}$$

The correlated data x' are plotted in the Hurst graph of Figure 7. This type of biasing introduces persistence.

A persistence observed in psychokinesis data on the basis of the above results could be associated with introduced biases correlating them collectively rather than sporadically. The nature of persistence in the data accumulated

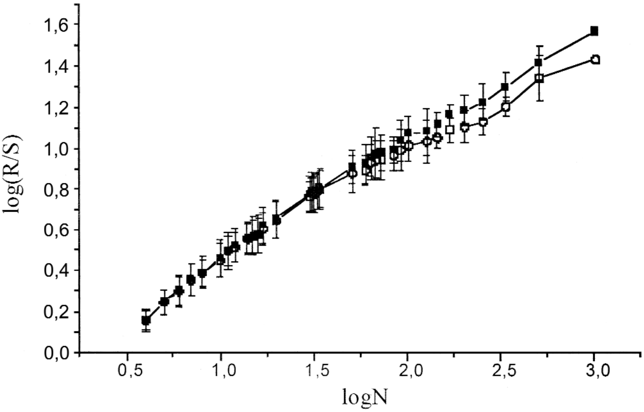

Fig. 6a.

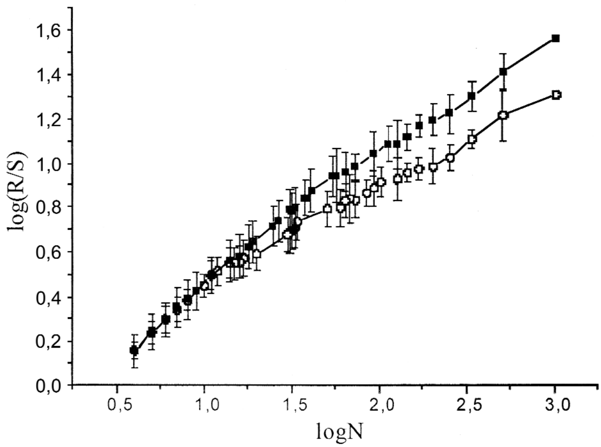

Fig. 6b.

Fig. 6. The Hurst graph of randomness test data (■) and biased randomness test (**O**) data by adding the number 50 (a) every 30 data points and (b) every 10 data points.

departure from the mean, implies a quality of a Jungian synchronicity (Jung, 1997) in the temporal sequences of random data. A synchronous trend can be consistent with absence of significant statistical deviations of averages from chance expectations. Random data may fluctuate about their theoretical aver-

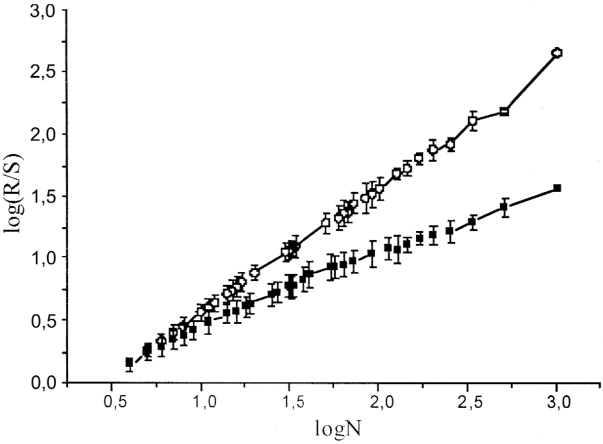

Fig. 7. The Hurst graph of randomness test data (■) and biased randomness test (○) data by correlating the data according to the relation (5.2).

ages to yield an overall non-statistically significant shift but may indicate, on the other hand, a tendency to cluster similar events in psychokinesis data in connection with the psychophysical parameter introduced.

6. Conclusions

It was shown that the rescaled range analysis done on sets of PK-RNG data provides an alternative source of information for tracing biases and investigating periodical features within the data. Most of the data presented here were found weakly biased having a trend of persistence in the accumulated departure from the mean, according to the fractional Brownian motion model, fBm. However, it was not obvious and conclusive, whether such weak biases were associated with the psychophysical parameter introduced in the PK tests or simply the result of chance fluctuations. It was shown, by introducing artificial biases in the data, that a persistent fBm indicates a data correlation in the whole of random units rather than a selective biasing of some of them.

Within its limitations the rescaled range analysis offers an alternative method for studying PK-RNG data. The information gained by its application may stimulate new models, give a deeper insight into the psychokinesis process and, more importantly, trigger new experimental techniques.

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