Journal of Scientific Exploration, Vol. 16, No. 1, pp. 117–124, 2002 0892-3310/02

Unification of the Physics of Interacting Bosons and Fermions Through (q, - q) Pair Correlation

YATENDRA S. JAIN

Department of Physics
N.E.H.U., Shillong-793 022, Meghalaya, India

Abstract—A brief qualitative analysis of widely different many body quantum systems is presented. It uses the framework of our new microscopic theory of a system of interacting bosons such as liquid ^4He, which is published in the current issue of this journal (pp. 77–115). The analysis includes dilute bose gases such as ^{87}Rb, low dimensional systems of interacting bosons, liquid ^3He, atomic nucleus, and superconductors. It concludes that the basic aspects of all these systems can be understood in terms of the Bose Einstein condensation of (**q**, - **q**) pairs. Our picture of superconductivity not only agrees with its BCS model but also helps in clarifying its several aspects.

Keywords: bosnic system — fermionic system — many body system — unification of theory

1.0: Introduction

Bose Einstein condensation (BEC) of particles (or pairs of particles) has been viewed[1] as a prime factor that can, presumably, unify our understanding of the unique properties of widely different *many body quantum systems* (MBQS), *viz.*, superfluids, superconductors, atomic nuclei, recently discovered BEC in systems like ^{87}Rb dilute gas[2], *etc.* While the nine important themes, that may help in understanding the association of BEC with low energy behavior of all such systems, have been listed by Snoke and Baym[3], the developments related to the crossover from fermionic to bosonic behavior and its converse as a basis for the unification of BEC and BCS models have been reviewed by Randeria[4], Ropke[5] and Hellmick *et al.*[6]. However, in a recent development[7,8] we succeeded in formulating the long awaited microscopic theory of a *system of interacting bosons* (SIB) such as liquid ^4He in terms of BEC of (**q**, - **q**) pairs. The similarity of this aspect of our theory with the accepted origin of superconductivity and superfluidity of fermion systems[9] in similar pairs (*i.e.,* Cooper pairs[10]) motivated us to make a priory qualitative analysis of widely different systems of interacting bosons and fermions to find that the condensation of (**q**, - **q**) pairs can really unify our understanding of the low energy behavior of all such systems. We hope to report a detailed quantitative analysis of these systems in near future.

Guided by the basics of our other studies[7,8], this qualitative analysis starts with a presumption that particles in all systems which may exhibit the phenomenon of superfluidity or superconductivity, move as free particles on a surface of constant negative potential. While the major component of inter-particle interaction $V(r)$ should be a short range r-dependent strong repulsion, the presence of a weak but slightly long range inter-particle attraction is necessary. The inter-particle phase correlation and the *wave packet* (WP) size, $\lambda/2 = \pi/q$, of particles play a very important role in deciding all unique properties of these systems attributed to the manifestation of quantum effects at macroscopic level[7,8]. The similarity of the trapped dilute gases, small clusters, atomic nuclei, *etc.,* with tiny drops of superfluid ^4He or ^3He stems from the fact that the nature of forces opposing the escape of a particle from the surface of a drop can also be considered to be closely harmonic.

2.0: BEC in ^{87}Rb Type Dilute Gasses

The nature of BEC in ^{87}Rb type dilute boson gases[2,11–14] at the initial stage[2], was believed to be a manifestation of $p = 0$ condensate in a *system of non-interacting bosons* (SNIB). However, this belief did not find its base when a wealth of experimental data on many such systems became available[14]. One finds that inter-particle interactions in these systems too play an important role in deciding the amount of $p = 0$ *condensate* ($n_{p=0}(T)$) conventionally believed to be the basis of *low temperature* (LT) behavior of a SIB[14]. As such the difference in the BEC state of trapped ^{87}Rb type dilute gases with superfluid ^4He is quantified by the difference in their $n_{p=0}(T)$ estimated to be ≈ 0.6[15] and ≈ 0.1[9], respectively. The new BEC state is now believed to be identical to that in *He-II* and the theories of the phenomenon have been developed by incorporating interactions[14]. However, since these theories[14] use conventional approach, one can not be sure of their accuracy at quantitative scale. On the other hand, since particles in both (i) the tiny drop of liquid ^4He and (ii) trapped dilute gases like ^{87}Rb are confined to within a specific volume by a *harmonic potential* (HP) and they do not occupy common coordinate in space, our theory can be applied to these cases identically. As such the interactions are equally important in deciding the properties of a dense SIB like liquid ^4He as well as a dilute SIB like ^{87}Rb gas. Further since our theory has successfully explained the properties of the former, we believe that it should, undoubtedly, be good enough to explain the properties of dilute gases too with high degree of accuracy, clarity and simplicity.

Applying the inference of our theory that all particles in the *ground state* (G-state) of the system represents equal size WPs, the G-state of N particles confined to a HP of average frequency $\tilde{\omega}$ would be one whose eigenfunction has $N^{1/3} + 1$ nodes or $N^{1/3}$ *anti-nodal regions* (AR) on each axis so that the state has a total of N AR to accommodate N particles. Since an eigenfunction of a state of a harmonic oscillator of quantum number ν has $\nu + 2$ nodes or $\nu +$ 1 anti-nodal regions, the ν of the desired state is $\nu = N^{1/3} - 1$ and this state corresponds to a de Broglie wave length

$$\lambda_v = h/\sqrt{2m\hbar\tilde{\omega}\cdot(\nu + 1/2)} \approx 2\pi\cdot\sqrt{\hbar/2m\tilde{\omega}}\cdot N^{1/3}. \quad (1)$$

Using $\lambda_v = \lambda_T = h/(2\pi m k_B T)^{1/2}$ (thermal de Broglie wave length), we can find $T = T_0$ corresponding to this λ_T. Because λ_v defines the G-state of the system, T_0 is an obvious equivalent of the zero point energy or the lower bound of T_c. We have

$$T_c \approx T_0 = \hbar\tilde{\omega}\cdot N^{1/3}/\pi\cdot k_B. \quad (2)$$

This agrees closely with

$$T_c^{\text{HP}} = \hbar\tilde{\omega}\cdot(N/1.202)^{1/3}/k_B \quad (3)$$

derived by de Groot *et al.*[16] for the temperature (T_c^{HP}) of BEC of N non-interacting particles in a HP. Following the argument behind Equation 19 of reference 8 for T_λ of a SIB, a more accurate T_c should be

$$T_c = T_0 + \frac{1}{4}\cdot T_c^{\text{HP}}. \quad (4)$$

Evidently, our theory concludes a T_c closely matching with T_c^{HP}.[16] The rest of the properties of these condensates should be as predicted by our theory[8].

By analogy with references 7 and 8, it may be inferred that the BEC in these gases occurs in a state of $q = \pi/d$ and $K = 0$ and particles in this state assume a close packed arrangement of their WPs and a locking of their relative ϕ-positions at $\Delta\phi = 2n\pi$. The state should exhibit macromolecular behavior and a kind of oneness of the system as envisaged respectively, by Foot and Steane[17], and Taubes[18], as well as the long range coherence, *etc.* The sample should exhibit collective oscillations as well as a single particle motion at wave vector $Q > 2\pi/\sigma$ (σ being the scattering length of particles). By using $\lambda_v = 2d$, in the relation for the first sound velocity $v_p = v_g = (\pi)^{1/2}h/2md$ (*cf.* Equation 28 of reference 8), we find that $v_p = v_g$ for $Q \approx 0$ collective motions can be obtained from

$$v_p = v_g = \sqrt{\frac{h\tilde{\omega}\cdot N^{1/3}}{m}}. \quad (5)$$

It may be noted that the first sound velocity estimated from experimental data to be (≈5 mm/sec)[14,19] agrees with our theoretical estimate from Equation 5. A detailed study of these systems in the framework of our theory[8] will be published elsewhere.

3.0 Low Dimensional Systems

A 1-D (or 2-D) system that could be realized in a laboratory is nothing but a 3-D system having 2 (or 1) dimension(s) reduced to the size of the order of the inter-particle separation d (or $\lambda/2$). Evidently, all q, K, r and R retain their 3-D character, in principle. However, when t, representing the diameter of the narrow channel or the thickness of the film, satisfies $t \approx \lambda/2 = \pi/q$, to a good approximation **K** and **R** have 1-D and 2-D character. The only important aspect in which these systems may differ from a bulk 3-D system is that their smallest dimension t controls q through $q \geq \pi/t$, if $d \geq t$ (d = nearest neighbour separation). However, if the particle number density ($n = N/V$) of the system is such that $d \leq t$, then below certain temperature T_c we can have a well connected chain of particle WPs in the case of 1-D system or a 2-D network of close packed arrangement of particle WPs in the case of 2-D system. Evidently, our theory should have no difficulty in accounting for the superfluid behavior for such 1-D and 2-D boson systems because, contrary to the conventional theories which presume that superfluidity arises when particles in the system have $\lambda \approx \infty$, our theory concludes that the phenomenon arises when all particles in the system satisfy $\lambda/2 = d$. While particles in a low dimensional system can not have $\lambda \approx \infty$ (characteristic of $p = 0$ condensate), they can certainly satisfy $\lambda/2 = d$ if $d \leq t$; here d and t are defined by excluding the solidified atomic layers on the surface of the channel or the substrate. If $t < d$, we can only have $\lambda/2 \leq t$, implying that particles can not have $\lambda/2 = d$ and the system can not exhibit superfluidity. In this regard we also find that the WPs of neighbouring particles fail to connect each other to develop a macroscopic chain (or 2-D network) of SMWs. Evidently, $d \leq t$ identified as the *connectivity condition* has to be satisfied by the system for superfluidity. In other words, n for a given t of 1-D and 2-D systems should, respectively, satisfy

$$n \geq t^{-1} \quad \text{and} \quad n \geq t^{-2} \tag{6}$$

Note that (i) the surface of the channels (or substrates), in contact with the sample atoms, used to realize 1-D (or 2-D) systems in the laboratory is normally rough at a scale of d, (ii) the diameter of the channels has fluctuations of the order of d, and (iii) particles moving near the substrate surface have increased effective mass because they experience surface potential of attractive nature. Naturally, the λ-point that largely depends on d through $\lambda_T = 2d$ in different parts of the sample occurs at different T_λ^{1D} (or T_λ^{2D}) $< T_\lambda$ of their bulk system because d in all such systems is always larger than the d in the bulk.

For a bulk system we note that the *temperature* (T) of maximum density (T_ρ), the T of λ-peak of specific heat (T_{sp}) and T of the onset of superfluidity (T_{sf}) compares with T_λ as T_ρ being slightly higher than T_λ, $T_{sp} = T_\lambda$ and $T_{sf} = T_\lambda$. However, these temperatures are not expected to be as sharp as for the bulk system because of possible difference in n (or d) in different parts of the 1-D and 2-D systems realized in the laboratory. Further, since the onset of superfluidity can be expected only when the process of transition is completed in the entire sample, T_ρ, T_{sp} and T_{sf} should compare as

$$T_\rho > T_{sp} > T_{sf} \tag{7}$$

which agrees with experiments[20]. Note that this analysis does not apply to those 1-D (or 2-D) systems in which 2 (or 1) dimension(s) are reduced to zero exactly. Such systems of theoretical possibilities are discussed at great length in references 21 and 22. As such the analysis presented in this report indicates that our theory[7,8] has great potential to understand the behavior of 1-D and 2-D systems that can be realized in the laboratory.

4.0 Fermi Systems

4.1: Liquid ^3He Type Fermi Systems

While suggesting the application of our approach to a fermi system, we assume that spin-spin interaction can be treated as a perturbation on the fermi particle states described by Equation 10 of reference 8. These states correspond to a hamiltonian in which fermi particles are treated as hard balls moving freely on a surface of constant negative potential. Using the right part of Equation 10 of reference 8 and the condition $\lambda/2 \leq d$, we note that: (a) all HC fermions in their G-state should define a close packed arrangement of WPs of equal size $\lambda/2 = d$ implying that their $q = q_0 = \pi/d$, and (b) particles have fermi distribution over the possible states of K representing the motion a free particle of mass $4m$ placed in a 3-D box. We naturally have

$$T_0 = \frac{h^2}{8\pi m k_B \cdot d^2} \quad \text{and} \quad T_F = \frac{h^2}{8\pi m k_B} \cdot \left(\frac{N}{1.5045V}\right)^{2/3}, \tag{8}$$

where T_0 and T_F, respectively, represent the T equivalent of zero-point energy (or momentum $q_0 = \pi/d$) and Fermi energy attributed to K motions of particles of mass $4m$; all notations have their usual meaning. Equation 8 renders T_0 = 1.79 K and T_F = 1.36 K for liquid ^3He. The system is expected to have two kinds of excitations: (i) single particle excitations which make a particle move from lower K level to higher K level, obviously above Fermi level and (ii) collective oscillations such as phonons and omons which may occur without any change in Fermi distribution of particles. SMW pairs of mass $2m$ should serve as the basic units of the system for these collective motions. The overlap of WPs and inter-particle attraction can produce a weak effect—a fall in q motion energy $\Delta E_f(T)$ in the same manner we have $E_g(T)$ fall in such energy of bosons (*cf.* Section 5.4 of reference 8). However, since the Pauli exclusion does not allow two fermions to have the same energy, the weak effect fails to show up unless the system has effective q motion degeneracy, *i.e.*, T falls below to T_c—the temperature equivalent of $\Delta E_f(0)$. This renders

$$k_B T_c = \Delta Ef(0), \tag{9}$$

which is almost identical to the well known relation between T_c and energy gap for superconductors[23]. Since the zero point repulsion, interatomic attraction and WP size in liquid ^3He are nearly equal (at least in their order of magnitude) to those for liquid ^4He, we can have $\Delta E_f(0) \approx E_g(0)$ *at least in order of its magnitude*. Using this observation and $E_g(0) \approx .142$ J/Mole for liquid ^4He (*cf.* Section 7.0 of reference 8) in Equation 9 we roughly estimate $T_c \approx 2.5$ mK for liquid ^3He which agrees with experimental value[24]. In summary the superfluidity in Fermi and Bose liquids is expected below a T of the order of gap energy ($\Delta E_f(0)$) and zero point energy (ε_0), respectively. We find that point (b) mentioned above can also explain why should the specific heat data of ^3He near T_c should reveal $m_F \approx 4m$[24].

4.2: Atomic Nucleus

In what follows from the above discussion, an atomic nucleus can be identified as a tiny drop of fermi superfluid where WPs representing different nucleons define a close packed system. These nucleons occupy ϕ-positions differing by $\Delta\phi = 2n\pi$. The nucleons cease to have random motion since they can move only in the order of their locations. As such the arrangement of nucleons in a tiny drop of superfluid resembles closely a solid-like structure. The nucleons follow fermi distribution over the states of K whose allowed values correspond to those of a free particle of mass $4m$ kept in a spherical shell of the size of the nucleus; the shell may have some deformations in its shape depending on its volume or related parameters. We believe that this picture should help in having a better understanding of the nucleus. The effects of the coupling between angular momenta, surface structure, finer details of inter-nucleon interactions, *etc*., on the allowed values of K can be worked out appropriately by considering the above stated close packed arrangement of nucleons. As such the nucleus can have (i) collective motions representing density oscillations in the nucleus without affecting the fermi distribution of nucleons in the states of momentum K, (ii) rotations, and (iii) single particle excitations involving changes in the distribution of nucleons over the states of K. We note that this picture has all the basic characteristics envisaged in partially successful complementary models: (i) single particle model, (ii) the shell model, (iii) the collective model, and (iv) the liquid drop model[25]. Naturally, our approach has the necessary potential to unify these models.

4.3: Superconductors

To understand the phenomenon of superconductivity, we note that the WP size of electrons increases with decreasing T, and finally at $T = T_c$ tends to cross the size of channels (diameter, d) available for their passage. This causes the WPs to overlap with the surrounding atoms producing electrical polarization and elastic strain in the lattice. This effect, serving as the source of interaction between electrons and phonons, does not differ from the one responsible for the formation of Cooper pairs. Evidently, our framework meets the starting point of BCS model[26] and the rest follows. However, our framework adds two important points to the BCS model.

First, the transition to superconducting state takes place when increasing WP size of conducting electrons pushes the lattice particles from their equilibrium positions decided by lattice forces. This process of lattice strain is obviously opposed by the lattice but in a new equilibrium the channel diameter increases locally from $d = d_c$ (a value of d at $T = T_c$) to $d = d_0$ (a value of d at $T = 0$). Consequently, the q motion energy of electrons falls from $h^2/8md_c^{\,2}$ to $h^2/8md_0^{\,2}$, *i.e.*,

$$\Delta E = E_{\text{gap}} \approx h^2(d_0 - d_c)/4\pi k_B m^* d_c^3 \approx k_B T_c. \tag{10}$$

E_{gap} is identified as an energy gap by an analogy of the argument behind the similar relation (Equation 33 of reference 8) obtained for the case of a SIB-like liquid ^4He. It is equated to $k_B T_c$ by the same argument that we used for Equation 9. Evidently, our framework relates T_c with $d = d_c$, the lattice strain $d_0 - d_c$, and the effective mass m^* of electron.

Second, our framework identifies the fact that the superconducting phase has lattice strain and this inference has, in fact, been observed experimentally[27]. The strain energy stored in the lattice comes, obviously, from quantum correlations among the charge carriers coming into existence at $T < T_c$ and it be identified as the source of the so called *virtual phonons* which in BCS theory are believed to mediate electron-phonon interaction even at $T = 0$, at which no real phonon exists.

While the explicit dependence of T_c on d and m^*, as seen from Equation 10, is not concluded by BCS theory, the facts that (i) λ-point of ^4He in narrow channels depends on d and m, and (ii) energy of the quantum states of a particle depends on the size of container and m, *etc.*, reveal that such dependence is obvious. Using $m^* = m_e$ (simple mass of an electron), some typical values of $d_c = 4.0\text{Å}$, $d_0 - d_c = 0.005\text{Å}$, we get $T_c = 21.7$ K, which represents a typical T_c. Broadly speaking, the combination of three parameters in the factor $(d_0 - d_c)/m^*d^3$ can easily help in accounting for the whole range of T_c (*i.e.*, 0 to about 125 K, observed for normal as well as high T_c superconductors). It appears that a detailed study of microscopic conditions within a superconductor that may render $m^* < m_e$ could be of great significance for understanding higher values of T_c. And in this context every possible source of interaction of charge carriers with lattice has an important role to play. Naturally, this suggestive analysis does not exclude the importance of any interaction (other than the phonon mediated attraction) in bound pair formation. As such our framework strongly indicates that the phenomenon of high T_c superconductivity needs not be different from that of normal superconductors.

5.0: Conclusion

The framework of our new approach used to develop the long awaited microscopic theory of liquid ^4He type bosonic systems is shown to have enough potential to account for the behavior of other bosonic systems such as BEC states of ^{87}Rb type dilute gases, low dimentional SIB, as well as similar fermionic systems such as liquid ^3He, supercoductors, atomic nucleus, *etc*. We find that (**q**, -**q**) pair condensation in the ground state of the system can unify the understanding of the physics of bosonic as well as fermionic systems. The ground state for a bosonic system has $q = q_0 = \pi/d$ and $K = 0$ and that of a fermionic system has $q = q_0 = \pi/d$ and K ranging from $K = 0$ to $K = K_F$. In both systems particles in their LT phases have ordered positions in phase space. While the superfluid transition in bosonic system occurs at a temperature equivalent of the ground state energy of a particle, in fermionic system it occurs at much lower temperature equivalent of per particle binding energy (*i.e.,* the energy gap).

Notes

1. Griffin, A., Snoke, D. W., & Stringari, A. (Eds.) (1995). *Bose Einstein Condensation*. Cambridge: Cambridge University Press.
2. Anderson, M. H., Ensher, J. R., Mathews, M. R., Wiechman, C. E., & Cornell, E. A. (1995). *Science, 269,* 198.
3. Snoke, D. W., & Baym, G. in reference 1, pp. 1–11.
4. Randeria, M. in reference 1, pp. 355–392.
5. Ropke, G. in reference 1, pp. 574–583.
6. Hellmich, A., Ropke, G., Schnell, A., & Stein, H. in reference 1, pp. 584–594.
7. Jain, Y. S. (1997). Cond-mat 9712150. (e-print at http://xxx.lanl.gov).
8. Jain, Y. S. (2002). Microscopic theory of a system of interacting bosons: A unifying new approach. *Journal of Scientific Exploration, 16,* 77–115.
9. Tilley, D. R., & Tilley, J. (1986). *Superfluidity and Superconductivity*. Bristol: Adam Hilger Ltd.
10. Cooper, L. N. (1956). *Physical Review, 104,* 1189.
11. Andrews, M. R., Kurn, D. M., Miesner, H. J., Durfee, D. S., Townsend, C. J., Inouye, S., & Ketterle, W. (1998). *Physical Review Letters, 79,* 553.
12. Davies, K. B., Mewes, M.-O., Andrews, M. R., van Druten, N. J., Durfee, D. S., Kurn, D. M., & Ketterle, W. (1995). *Physical Review Letters, 75,* 3969.
13. Bradley, C. C., Sackett, C. A., Tollett, J. J., & Hulet, R. J. (1995). *Physical Review Letters, 75,* 1687.
14. Dalfovo, F., Giorgini, S., Pitaevskii, L. P., & Stringari, S. Cond-mat 9806038 V2. (e-print at http://xxx.lanl.gov).
15. Ensher, J. R., Jin, D. S., Matthews, M. R., Weiman, C. E., & Cornell, E. A. (1996). *Physical Review Letters, 77,* 4984.
16. de Groot, S. R., Hooyman, G. J., & ten Seldam, C. A. (1950). *Proceedings of the Royal Society of London A, 203,* 266.
17. Foot, C. J., & Steane, A. M. (1995). *Nature, 376,* 213.
18. Taubes, G. (1995). *Science, 269,* 152.
19. Cornell, E. A. (1997). Private communication.
20. Eggington. A. (1975). In Armitage, J.G.M., & Ferquhar, I. E. (Eds.). *The Helium Liquids* (p. 195). London: Academic Press.
21. (a) Girardeau, M. (1960). *Journal of Math. Physics, 1,* 516. (b) Lieb, E. H., & Liniger, W. (1963). *Physical Review, 130,* 1605. (c) Yang, C. N. (1967). *Physical Review Letters, 19,* 1312.
22. Kosterlitz, J. M., & Thouless, D. J. (1973). *Journal of Physics, C6,* 1181.
23. Feynman, R. P. (1976). In *Statistical Mechanics* (p. 284). Reading, MA: W. A. Benjamin.
24. Wheatley, J. C. (1975). In Armitage, J.G.M., & Ferquhar, I. E. (Eds.). *The Helium Liquids* (p. 260). London: Academic Press.
25. Preston, M. A. (1962). *Physical of Nucleus*. Reading: Addison Wesley.
26. Bardeen, J., Cooper, L. N., & Schrieffer, J. (1957). *Physical Review, 108,* 1175.
27. Ranninger, J. in reference 1, pp. 393–417.