Can Longitudinal Electromagnetic Waves Exist?

GERHARD W. BRUHN

Darmstadt University of Technology,
Department of Mathematics, Darmstadt, Germany
e-mail: bruhn@mathematik.tu-darmstadt.de

Abstract—In discussions on electro smog K. Meyl has proposed to consider the "dangerous" scalar waves (1) in addition to Hertzian waves. But we have already shown in a previous paper (2) that, indeed, Meyl's scalar waves cannot cause any harm, to anybody—since *they do not exist.* Some readers have interpreted Meyl's scalar waves to be identical with longitudinal electromagnetic waves, but this is not clear due to Meyl's inconsistencies; e.g., his splitting the wave equation is erroneous. Therefore, to calm down our worried readers, below we shall prove that longitudinal electromagnetic waves are harmless as well by recalling a well-known classical result: Plane longitudinal electromagnetic waves *do not exist.* We supplement this by showing that longitudinal spherical electromagnetic waves have the same pleasant property: *They don't exist.*

Keywords: electromagnetism

Introduction

We consider a homogeneous medium of constant dielectricity ϵ and constant permeability μ that is free of electric sources and currents. Then all electromagnetic processes are governed by the homogeneous Maxwell equations (abbreviation: $\partial_t = \partial/\partial t$)

$$\text{curl } \mathbf{E} = -\mu\partial_t\mathbf{H}, \tag{1}$$

$$\text{div } \mathbf{E} = 0, \tag{2}$$

$$\text{curl } \mathbf{H} = \varepsilon\partial_t\mathbf{E}, \tag{3}$$

$$\text{div } \mathbf{H} = 0. \tag{4}$$

From these equations it can be derived by standard elimination processes that both the electric and the magnetic vector **E** and **H** fulfill the wave equations

$$\Delta\mathbf{E} - c^{-2}\partial_t^2\mathbf{E} = \mathbf{0} \quad \text{and} \quad \Delta\mathbf{H} - c^{-2}\partial_t^2\mathbf{H} = \mathbf{0}, \quad \text{where } c^{-2} = \varepsilon\mu. \tag{5}$$

The wave equation admits so-called *plane wave* solutions of the type

$$\mathbf{E}(\mathbf{x},t) = \mathbf{E}_0\, e^{i\omega(t-x/c)} \quad \text{and} \quad \mathbf{H}(\mathbf{x},t) = \mathbf{H}_0\, e^{i\omega(t-x/c)}. \tag{6}$$

Here the electromagnetic state is propagating in a fixed direction, the direction of the x-axis in our example. The factor $e^{i\omega(t-x/c)}$ is responsible for the oscillation with the constant circular frequency ω. The constant vector factors $\mathbf{E}_0$ and $\mathbf{H}_0$ are the *amplitude* factors of the wave.

The wave equations (Equation 5) do not restrict these amplitude factors. So one can chose the factors $\mathbf{E}_0$ and $\mathbf{H}_0$ parallel to the direction of the wave propagation, i.e., the direction of the x-axis. In this case the wave is called (oscillating) *longitudinal*. In applications in acoustics and waves in elastic media such longitudinal waves occur.

Longitudinal waves in elastic media were already well known when the theory of electromagnetism was created by Faraday, Œrsted, Ampere, Maxwell, et al. in the 19th century. The analogies between electromagnetism and elasto-mechanics were intensively discussed, and it was a great surprise when they discovered during further development that electromagnetic plane waves couldn't be longitudinal.

This result is a consequence of the fact that electromagnetic waves have to fulfill not only the wave equations (Equation 5) as the elasto-mechanical waves do but also the Maxwell equations (Equations 1–4). And this latter fact is much more restrictive. Ignoring this fact is one of Meyl's flaws (1).

The reader can learn this result from A. Sommerfeld's classical book on electrodynamics (3), p. 36, but we shall give an independent proof here.

For obtaining the claimed result we merely have to insert the solutions (Equation 6) of the wave equations into the Maxwell equations (Equations 1–4).

First we remark that time derivation $\partial_t = \partial/\partial t$ of Equation 6 yields the equations

$$\partial_t\, \mathbf{E} = i\omega\, \mathbf{E} \quad \text{and} \quad \partial_t\, \mathbf{H} = i\omega\, \mathbf{H}. \tag{7}$$

Then we remember the definition of curl by the DEL operator ∇ (cf. Spiegel (4), p. 57 f.). Hence we obtain, since the amplitude vectors $\mathbf{E}_0$ and $\mathbf{H}_0$ are constant,

$$\text{curl}\, \mathbf{E} = \nabla\times(\mathbf{E}_0\, e^{i\omega(t-x/c)}) = (\nabla\, e^{i\omega(t-x/c)})\times\mathbf{E}_0$$

and

$$\text{curl}\, \mathbf{H} = \nabla\times(\mathbf{H}_0\, e^{i\omega(t-x/c)}) = (\nabla\, e^{i\omega(t-x/c)})\times\mathbf{H}_0.$$

Using $\nabla\, e^{i\omega(t-x/c)} = -i\omega/c\, e^{i\omega(t-x/c)}\, \mathbf{e}_1$ ($\mathbf{e}_1$ = unit vector with direction of the x-axis), we can convert these equations to

$$\text{curl } \mathbf{E} = -i\omega/c \; \mathbf{e}_1 \times \mathbf{E} \quad \text{and} \quad \text{curl } \mathbf{H} = -i\omega/c \; \mathbf{e}_1 \times \mathbf{H}. \tag{8}$$

Hence, by inserting the Equations 8 into the Maxwell equations (Equations 1 and 3) we obtain

$$1/c \; \mathbf{e}_1 \times \mathbf{E} = \mu \mathbf{H} \quad \text{and} \quad -1/c \; \mathbf{e}_1 \times \mathbf{H} = \varepsilon \mathbf{E}.$$

Since the cross product is perpendicular to each of its factors, we may conclude

$$\mathbf{E} \perp \mathbf{e}_1 \quad \text{and} \quad \mathbf{H} \perp \mathbf{e}_1. \tag{9}$$

Thus we have the result that the electric and magnetic oscillations of a plane electromagnetic wave are always *perpendicular* to its direction of propagation. Waves of that type are called *transversal.* **Longitudinal plane electromagnetic waves are impossible.**

We complete our calculation by checking the Maxwell equations (Equations 2 and 4). From Equation 6 we obtain

$$\begin{aligned} \text{div } \mathbf{E} &= \nabla \cdot (\mathbf{E}_0 \; e^{i\omega(t-x/c)}) = \mathbf{E}_0 \cdot (\nabla e^{i\omega(t-x/c)}) \\ &= -i\omega/c \; \mathbf{e}_1 \cdot \mathbf{E} = 0 \quad \text{due to } \mathbf{E}_0 \perp \mathbf{e}_1. \end{aligned}$$

The evaluation of div **H** runs analogously. Hence under the conditions (Equation 9), Equation 6 represents solutions of the Maxwell equations (Equations 1–4).

Finally a remark to the general case: *General* electromagnetic waves may be much more complicated than the special types (Equation 6) considered above. But nevertheless one can make out a propagation direction: The direction of energy transport of the wave, which is given by the Poynting vector

$$\mathbf{S} || \mathbf{E} \times \mathbf{H}, \tag{10}$$

where we have to insert *real* vector solutions (**E**, **H**) of the Maxwell equations (Equations 1–4). But from Equation 10 we may conclude that even in the general case electromagnetic waves are oscillating *perpendicular* to their direction of energy transport: In this sense we may formulate the **general** result:

Solutions (**E**, **H**) of the Maxwell equations are always *transversal.* **Longitudinal waves are impossible.**

References

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