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BOOK REVIEW

Where Mathematics Comes from: How the Embodied Mind Brings Mathematics into Being by George Lakoff and Rafael E. Núñez. New York: Basic Books, 2000. 512 pp. $29.99. ISBN 978-0465037711.

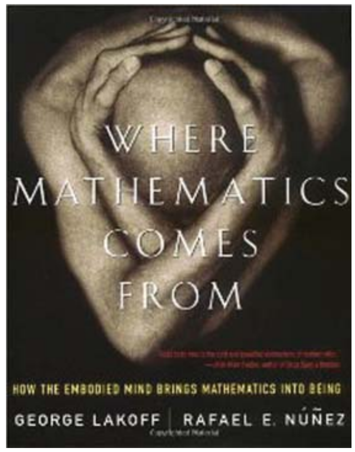
This book announces a new academic discipline, the "cognitive science of mathematics" (p. XI), by demonstrating how an empirical examination of the *ideas* underlying our use of mathematical symbols and calculations must employ metaphors grounded in the "embodied mind." The authors ruthlessly attack what they dub "The Romance of Mathematics" (p. XV), their metaphor for any approach that treats mathematics as grounded in an abstract, disembodied yet objective reality that mysteriously provides the essential structure to the natural, human world.

The authors declare their a priori assumptions in the Introduction, the most essential being that "human mathematics . . . is an empirical scientific question, not a mathematical or a priori philosophical question" (p. 3). Solely on this basis can they claim that cognitive science, *and cognitive science alone*, answers the question posed by the book's title. Repeatedly referring to this and other central claims as "arguments," the authors actually take as their foundational *presupposition* that "whatever 'fit' there is between mathematics and the world occurs in the minds of scientists" (p. 3). Their rejection of "Platonic mathematics" (p. 4) is obviously circular: The conclusion affirms what the first premise assumed, that "transcendent Platonic mathematics" cannot be "human mathematics." (This tendency toward circularity pervades the book, as when the authors conclude that mathematics, which they have assumed is necessarily grounded in "the cognitive apparatus" [p. 30], turns out to be "not independent" of that apparatus.) The authors never acknowledge that *anything* transcendent, once it is made known to us, must (by definition) make use of metaphors and/or other symbolic processes; so the presence of such metaphors cannot disprove an original transcendence. Following this dubious starting point, an irony colors the entire book: After making such a concerted effort to debunk Platonic (transcendent) mathematics, by demonstrating how all mathematical truths require metaphors in order to be understood by humans, their demonstration could also explain how *Platonic* mathematics comes to be known by us! *This* issue, the question of whether or not there *is* a transcendent realm wherein mathematical truth might be said to "exist," is entirely philosophical; as such, it necessarily transcends this book's empirical, scientific mandate.

The book's 16 chapters are grouped into five parts, with a sixth part reporting a four-part case study. Chapters 1–4, on the Embodiment of Basic Arithmetic, contain little original research, being primarily a summary of some basic tools of "second generation" cognitive science, grafted into reports on the findings of various scientists, regarding the innate abilities of human beings (and some animals) to count up to small numbers (or "subitize") and perform various other simple mathematical functions. My review of one of this book's primary sources, Debaene's *The Number Sense* (also the likely answer to the question: Where did the authors' *title* come from?), is being published together with the present review and covers a similar range of empirical issues, so I shall not comment further on such details here.

The authors sometimes attribute cognitive mechanisms to the brain too hastily (without evidence or argument): e.g., they state that a type of *metaphor*, associating simple arithmetic with "object collection," "arises naturally in our brains" (p. 60; cf. p. 78). But do metaphors themselves really exist *in the brain*? Likewise, they claim arithmetic *arises out of* the "4Gs" (four "grounding metaphors for arithmetic")—The Zero Collection Metaphor (p. 64), The Zero Object Metaphor (p. 67), The Measuring Stick Metaphor (p. 68), and Arithmetic as Motion Along a Path (p. 71)—yet they offer nothing close to an argument that would prove these metaphors are necessary for the very *possibility* of human arithmetic. Provided we remember that the authors' approach is descriptive, the high-sounding conclusions regarding their claims to have shown where arithmetic "comes from" can be accepted for what they are: *empirical* truth-claims. The 4Gs may correctly identify the empirical factors that "make the arithmetic of natural numbers natural" (p. 92) and thus be "constitutive of our fundamental understanding of arithmetic" (p. 94). Yet this does not prove that mathematics has no independent, transcendent status—any more than explaining how one learns the concept "God" would prove that no divine being exists outside of human religious traditions. The authors ignore the distinction that begins Kant's *Critique of Pure Reason*: The fact that "all our knowledge begins with experience" does not imply that "all our knowledge arises out of experience." Conflating "arithmetic" with "our understanding of arithmetic" (e.g., pp. 94–95), they assume that by uncovering the cognitive roots of the latter they are answering the "big question" of the former's source.

The same error pervades Part II, where the authors wade into the



deeper mathematical waters of algebra, logic, and set theory. They assume the task of discovering "what algebra *is*, from a cognitive perspective" (p. 119) exhausts the question of where algebra itself *comes from*. Likewise, they reduce Boolean logic and set theory to expressions of mathematical metaphor. They declare mathematical concepts such as "the universal class" to have no "objective existence at all," but to be "created" by an underlying metaphor (p. 131), and insist that "rules of inference" can "preserve truth" only because of such metaphors. Here, as throughout the book, the explanations are often intriguing, yet also frustratingly repetitious.

Whereas most publications in the Lakoff school of cognitive science are filled with an impressive array of metaphors, the empirical observations and scientific conclusions advanced in this book all stem from one foundational metaphor (p. 161), presented in various forms. Only in Part III (The Embodiment of Infinity) does this Basic Metaphor of Infinity (BMI) (pp. 8f.) come to the fore. Chapter 8 unpacks its structure by elucidating "the embodied source of the idea of infinity" in human "*aspectual systems*" (p. 155). A consistent defect throughout their discussion is the tendency to make ontological claims (e.g., "Wherever there is infinite totality, the BMI is in use" [p. 175]), when their argument only justifies *epistemological* claims regarding the necessity of metaphor for *human reference systems*. Chapters 9–11 persuasively argue that our use of real numbers, transfinite numbers, and infinitesimals also relies on the BMI.

Part IV presents a series of historically based accounts of The Discretization Program That Shaped Modern Mathematics (p. 257f.). As intellectual history, their claims about the centrality of metaphor in the various paradigm shifts they analyze may be questionable; but they undoubtedly expose numerous fascinating tendencies, highlighting some insightful implications for the way mathematics ought to be taught.

Part V concludes the main text by discussing "Implications for the Philosophy of Mathematics" (p. 335f.). Here the subtle circularity of the authors' overall argument comes to the fore, as they claim to have dismissed the possibility of any mathematical entities actually existing. In argument after argument they assume that the metaphorical structure of all human thought processes proves that things (e.g., numbers) in themselves are impossible. But what they attack is a straw man: the (silly) claim that numbers somehow exist “out in space” (p. 345).

Taken as an extended exercise in what the authors aptly describe as “mathematical idea analysis” (p. 29 et al.), this book is a tour de force for cognitive science. Despite the authors’ reluctance to acknowledge any predecessors, it defends an essentially Kantian thesis: The human mind itself *creates* mathematics. Yet the authors badly err by inferring from their own self-confessed presuppositions that their analysis *proves* mathematics to be *nothing but* “a mere historically contingent social construction” (p. 9). By taking this (unnecessary) extra step, mysteriously insisting—as if it were a magical incantation!—that “[t]here is nothing mysterious, mystical, magical, or transcendent about mathematics” (p. 377), they ironically end up defending an entirely non-empirical thesis that might be called *the Romance of the Cognitive Science of Mathematics*.

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